

SNIMP

Simplified NIM Play

SNIMP stands for “Simplified NIM Play”. As the name suggests this is a simplified version of the classical NIM game. SNIMP is based on the same underlying principle as NIM.

NIM is a very popular game, and the winning strategy for NIM is also quite well known. So why SNIMP?

Though the winning strategy for NIM is quite well known, I have seen students struggle to understand how the strategy really works behind the scenes, especially when the number of heaps is more than two. Students are able to apply the strategy easily and win the games comfortably, but the question still lingers in their mind “How does it work?”

In my quest to make the strategy become more obvious to the students, I ended up creating a simplified version of this game, and called it SNIMP.

SNIMP is a paper and pencil game with a set of simple rules that would be explained in the next section. Once a player gets used to the game, the winning strategy becomes obvious to the player. The player would be able to appreciate why and how the strategy works. And with this background, I will show how the SNIMP game can be extended into the classical NIM game with just one minor change in the setup. And ahoj, not only does the same winning strategy works, but also the strategy becomes more obvious to the player.

So without much further ado, let’s get into SNIMP.

SNIMP is a two player game which begins with a set of two or more numbers such that in each number, there are no repeating digits.

Ex: 31256 is a valid number, while 543123 is not since the digit “3” repeats.

Players play turn by turn, and on their turn, a player

- (a) Must remove one or more digits from any one number only.
- (b) Optionally add one or more digits back to the same number such that
 - a. Each added digit is lesser in value than any one of the removed digits.
 - b. No digits repeat within a number.

The player who makes the last move, to eliminate the last remaining number, is the Winner.

Note: In all the illustrations below, we will assume that Player-1 knows the strategy and plays according to the winning strategy.

Let us play SNIMP with two numbers.

By Kiran Bacche – Author of “The Magic and Joy of Exploding Dots” and “Mathematical Approach To Puzzle Solving”, Global Math Project Ambassador

Game 1: The initial set of numbers are 361 and 125

Moves	Turn	Number-1	Number-2	Comments
		361	125	Initial set of numbers
Move-1	Player-1	251	125	Remove 3 and 6, and add 2 and 5 into the first number
Move-2	Player-2	31	125	Remove 2 and 5, and add 3 into the first number
Move-3	Player-1	31	13	Remove 2 and 5, and add 3 into the second number
Move-4	Player-2	31	12	Remove 3, and add 2 into the second number
Move-5	Player-1	21	12	Remove 3, and add 2 into the first number
Move-6	Player-2	1	12	Remove 2 from the first number
Move-7	Player-1	1	1	Remove 2 from the second number
Move-8	Player-2	1	-	Remove 1 from the second number thereby eliminating the second number
Move-9	Player-1	-	-	Remove 1 from the first number thereby eliminating the first number. And thus crowned the winner.

Once you have gone through this, it would have become fairly obvious what the winning strategy is. Let us play one more game.

Game 2: The initial set of numbers are 29 and 371

Moves	Turn	Number-1	Number-2	Comments
		29	371	Initial set of numbers
Move-1	Player-2	8765431	371	Remove 2 and 9, and add 8, 7, 6, 5, 4, 3 and 1 into the first number.
Move-2	Player-1	731	371	Remove 8, 6, 5 and 4 from the first number
Move-3	Player-2	731	35421	Remove 7, and add 5, 4 and 2 into the second number
Move-4	Player-1	54231	35421	Remove 7, and add 5, 4 and 2 into the first number
Move-5	Player-2	1	35421	Remove 4, 3, 2 and 1 from the first number
Move-6	Player-1	1	1	Remove 4, 3, 2 and 1 from the second number
Move-7	Player-2	1	-	Remove 1 from the second number thereby eliminating the second number
Move-8	Player-1	-	-	Remove 1 from the first number thereby eliminating the first number. And thus crowned the winner.

So the winning strategy should be quite clear now. And can be summarized as below.

The strategy is to make sure that each digit appears exactly twice across the two numbers once your move is made. Since the digits cannot repeat within any number, the strategy for the two number SNIMP can be further simplified. On your move, you just have to make sure each number contains the same set of digits (order doesn't matter). This will ensure that whatever move you

By Kiran Bacche – Author of “The Magic and Joy of Exploding Dots” and “Mathematical Approach To Puzzle Solving”, Global Math Project Ambassador

opponent makes with one number, you can make an identical move with the other. So you will always have the chance to make the final move, and become the winner.

Now let’s turn our attention to SNIMP with three numbers. We will go through couple of examples, and the strategy would become obvious very quickly.

Note: In all the examples below, we will assume that Player-1 knows the strategy and plays according to the winning strategy.

Let us play SNIMP with three numbers.

Game 3: The initial set of numbers are 172, 386 and 3764

Moves	Turn	Num-1	Num-2	Num-3	Comments
		172	386	3764	Initial set of numbers
Move-1	Player-1	172	34126	3764	Remove 8, and add 4, 1 and 2 into the second number
Move-2	Player-2	2	34126	3764	Remove 1 and 7 from the first number
Move-3	Player-1	2	34126	3164	Remove 7, and add 1 into the third number
Move-4	Player-2	2	3	3164	Remove 4126 from the second number
Move-5	Player-1	2	3	32	Remove 6, 4 and 1, and add 2 into the third number
Move-6	Player-2	2	3	12	Remove 3, and add 1 into the third number
Move-7	Player-1	2	1	12	Remove 3, and add 1 into the second number
Move-8	Player-2	1	1	12	Remove 2, and add 1 into the first number
Move-9	Player-1	1	1	-	Remove 1 and 2 from the third number thereby eliminating it
Move-10	Player-2	-	1	-	Remove 1 from the first number thereby eliminating it
Move-11	Player-1	-	-	-	Remove 1 from the second number thereby eliminating the second number. And thus crowned the winner.

Once you have gone through this, you would have started to feel the winning strategy. Let us play one more game.

By Kiran Bacche – Author of “The Magic and Joy of Exploding Dots” and “Mathematical Approach To Puzzle Solving”, Global Math Project Ambassador

Game 4: The initial set of numbers are 91, 81 and 71

Moves	Turn	Num-1	Num-2	Num-3	Comments
		91	81	71	Initial set of numbers
Move-1	Player-2	87654321	81	71	Remove 9, and add 8, 7, 6, 5, 4, 3 and 2 into the first number
Move-2	Player-1	87	81	71	Remove 6, 5, 4, 3, 2 and 1 from the first number
Move-3	Player-2	87	7654	71	Remove 8 and 1, and add 7, 6, 5 and 4 into the second number
Move-4	Player-1	6541	7654	71	Remove 8 and 7, and add 6, 5, 4 and 1 into the first number
Move-5	Player-2	6541	1	71	Remove 7, 6, 5 and 4, and add 1 into the second number
Move-6	Player-1	6541	1	654	Remove 7 and 1, and add 6, 5 and 4 into the third number
Move-7	Player-2	1	1	654	Remove 6, 5 and 4 from the first number
Move-8	Player-1	1	1	-	Remove 6, 5 and 4 from the third number thereby eliminating it
Move-9	Player-2	-	1	-	Remove 1 from the first number thereby eliminating it
Move-10	Player-1	-	-	-	Remove 1 from the second number thereby eliminating the second number. And thus crowned the winner.

You should have now understood the strategy which happens to be exactly the same as in SNIMP with two numbers. The strategy is to make sure that each digit appears exactly twice across the three numbers at the end of your move. We will now quickly see how and why the strategy works for the 3-number SNIMP.

Using the strategy repeatedly, the three number SNIMP game will eventually reduce to the configuration {1, 12, 2} where 1, 12 and 2 are the three numbers remaining on your opponent's turn. From this situation, only the following moves are possible for your opponent.

Possible Moves by your Opponent	Your Next Move	Comment
{12, 2}	{2, 2}	This reduces to a 2 Number SNIMP Winning Position
{1, 2}	{1, 1}	
{1, 12}	{1, 1}	
{1, 1, 2}	{1, 1}	
{1, 12, 1}	{1, 1}	

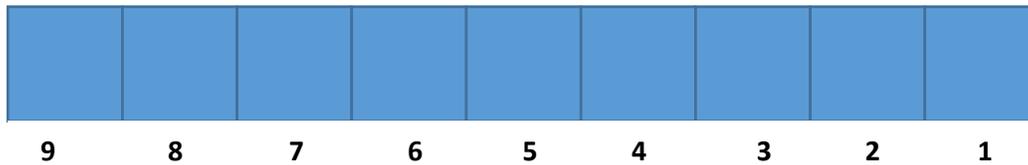
By Kiran Bacche – Author of “The Magic and Joy of Exploding Dots” and “Mathematical Approach To Puzzle Solving”, Global Math Project Ambassador

For any of these 5 possible moves, you can make the next move such that the game gets reduced into a 2-number SNIMP winning position. Easy, isn't it?

Note: There could be cases, where the game can get reduced to 2-number SNIMP winning position without necessarily going through the {1, 12, 2} configuration as shown in Game 4.

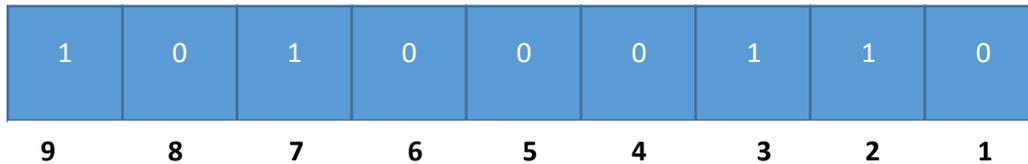
Now that we have understood how the strategy works for SNIMP, let us look at the game more closely from a modelling perspective, and make one small change in the setup.

We can model the numbers in SNIMP as a set of nine boxes whose values are 9, 8, 7 and so on.



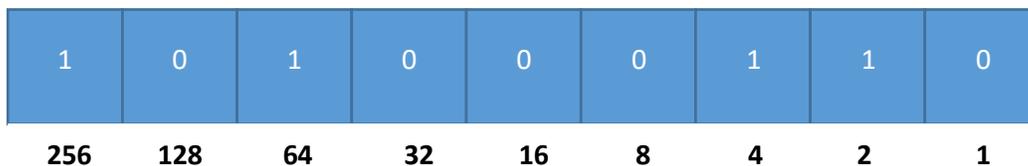
Every number in SNIMP is a set of these boxes with each box having either a 1 or a 0.

So for example, the number 9732 can be modelled as



As you can easily see, each box can hold either a 0 or a 1. Or in other words, a binary digit.

Let us make one small change in the setup. Instead of using the symbols 9, 8, 7 and so on for the value of the boxes, let us use a different set of symbols as shown below.



Ahoy, what do we have here? A binary number system! The value of the boxes are nothing but the place-value of the binary digits in this binary representation.

And the above model represents the number “ $256+64+4+2$ ” = 326

Unlike in the original SNIMP where we cannot choose numbers like 311 or 987655 (since the symbols repeat), in this new setup, we can choose any number since it can be converted to a binary representation where the new set of symbols (256, 128, 64, etc.) don't repeat.

By Kiran Bacche – Author of “The Magic and Joy of Exploding Dots” and “Mathematical Approach To Puzzle Solving”, Global Math Project Ambassador

Ex: 311 can be now chosen since it can be converted into “256:32:16:4:2:1”. (Instead of using the binary digits 0 and 1, we are using the place-value symbol of the binary digits that are set to 1). As you can observe the symbols 256, 32, 16, 4, 2 and 1 occur only once and do not repeat.

1	0	0	1	1	0	1	1	1
256	128	64	32	16	8	4	2	1

The rules for the modified SNMIP game (with the new setup) doesn't change.

Players play turn by turn, and on their turn, a player

- (a) Must remove one or more digits from any one number only.
- (b) Optionally add one or more digits to the same number such that
 - a. Each added digit is lesser in value than any one of the removed digits.
 - b. No digits repeat within a number.

Note: Here digit actually refers to the place value symbols “128”, “64”, and so on.

In this newer representation, 311 would get represented as “256:32:16:4:2:1”, 10 would get represented as “8:2” and so on. Adding all the digits/symbols of a number in the above representation actually gives the decimal value of the number. Removing higher value digits/symbols from a number and adding lower value digits/symbols into the same number basically means reducing the number to a smaller value. So representing the numbers in decimal form, the rules of the modified SNMIP game can be simplified as below.

Players play turn by turn, and on their turn, a player

- (a) Selects any one number and reduces it to a smaller number ≥ 0 .
- (b) Once a number becomes 0, it is considered eliminated.

The player who makes the last move, to eliminate the last remaining number, is the Winner.

And knowing the original SNIMP, you can see how the same strategy can be easily understood and applied to win the modified SNIMP by converting the decimal numbers into their binary representation.

The modified SNIMP game is nothing but the Classical NIM we all know and love to play!

SNIMP and NIM can be played with 4 or 5 numbers too. In a 4-number and 5-number SNIMP/NIM, a digit/symbol can occur four to five times across all the numbers. So what would be the winning strategy? **Clue:** Think on the same lines since the strategy is exactly similar.

Once you have figured out the strategy, it is time for a 10-number SNIMP and NIM. Are you game?

Enjoy and all the best!!